

Exercise 28

- (a) If $f(x) = e^x \cos x$, find $f'(x)$ and $f''(x)$.
- (b) Check to see that your answers to part (a) are reasonable by graphing f , f' , and f'' .

Solution

Calculate the first derivative of the given function.

$$\begin{aligned} f'(x) &= \frac{d}{dx}(e^x \cos x) \\ &= \left[\frac{d}{dx}(e^x) \right] \cos x + e^x \left[\frac{d}{dx}(\cos x) \right] \\ &= (e^x) \cos x + e^x(-\sin x) \\ &= e^x(\cos x - \sin x) \end{aligned}$$

Now calculate the second derivative.

$$\begin{aligned} f''(x) &= \frac{d}{dx}[f'(x)] \\ &= \frac{d}{dx}[e^x(\cos x - \sin x)] \\ &= \left[\frac{d}{dx}(e^x) \right] (\cos x - \sin x) + e^x \left[\frac{d}{dx}(\cos x - \sin x) \right] \\ &= (e^x)(\cos x - \sin x) + e^x(-\sin x - \cos x) \\ &= -2e^x \sin x \end{aligned}$$

The function and its first two derivatives are plotted below versus x .

